

**Teilprojekt B3.07**

# **Topologically Protected Circuits for Metrology**

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## 1 Charge solitons

In the last decade great progress has been achieved in observation and utilization of quantum phenomena in Josephson junction circuits. Macroscopic quantum effects have been realized based on the coherent properties of superconductors. At present the main activities in this field are focused on optimizing qubit circuits and on the realization of a quantum information processor. In this project we focus on a different subject. We study the possibility of using the collective excitations in arrays of Josephson elements for metrological applications with high level of precision. In particular the aim is to improve the accuracy of frequency-current conversion towards a metrological current standard.

From the start of the project (July 2009) we have mainly focused on the subject of charge solitons in gated arrays of Josephson junctions. Such solitons are dual objects to the well-understood flux solitons in long Josephson junctions. They carry exactly the charge of one Cooper pair. The solitons are controlled by the gate voltages in the array and can be, in principle, launched and stopped at will. A charge soliton is a mesoscopic object with size extending over several superconducting islands. Thus, the influence of the background charge fluctuations and trapped charges should be reduced compared to conventional single-electron devices. All these features should allow for very precise frequency to current conversion. Yet, the charge solitons have never been observed in the ballistic regime. Also the controversy remains about the nature of the kinetic mass of a charge soliton. Thus, we have concentrated on the clarification of the soliton mass controversy and further developed theory of charge solitons in 1D arrays. In the future we will focus on controlling the solitons precisely enough so that metrological application become feasible.

The research is being performed in close collaboration with the groups of A. Ustinov and of R. Schäfer [B3.6] who study the dissipative transport of charge solitons experimentally. These groups have accumulated considerable experience in dealing with flux solitons in long Josephson junction and in nanofabrication.

### 1.1 Many-body tight-binding method

In [B3.7:1] and [B3.7:2] we have developed a new tight-binding method which allowed us to investigate the dispersion relation of charge solitons in Josephson arrays (Fig. 1). In particular we have identified an interesting regime of "small charge solitons" (polarons). In this regime the charge dynamics is strongly influenced by the polaronic effects, i.e., by dressing of a Cooper pair by charge dipoles.

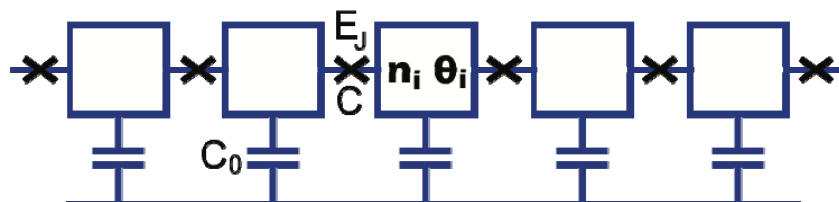


Fig. 1: Josephson Array. The superconducting islands are connected by Josephson junctions characterized by Josephson energy  $E_J$  and by capacitance  $C$ . The gate capacitances  $C_0$  determine the electrostatic screening length of the array  $\Lambda=(C/C_0)^{1/2}$ .

We have identified the relevant charge configurations, which take part in the formation of the charge solitons (Fig. 2). In order to construct the many-body tight-binding Hamiltonian we developed a spin-1/2 representation of the relevant charge configurations. This formalism has allowed us to take into account about 1000 charge states.

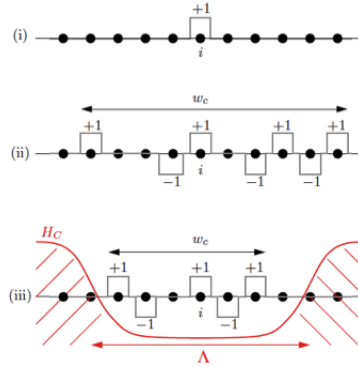


Fig. 2: Relevant charge states. The total charge of all states is equal one. The “width” of the relevant configurations is limited by the screening length  $\Lambda$ .

Numerical diagonalization of the tight-binding matrix produced dispersion relations for the charge solitons (Fig. 3) from which we were able to determine the effective mass. Another interesting effect is the flattening of the dispersion relation in the outer part of the Brillouin zone.

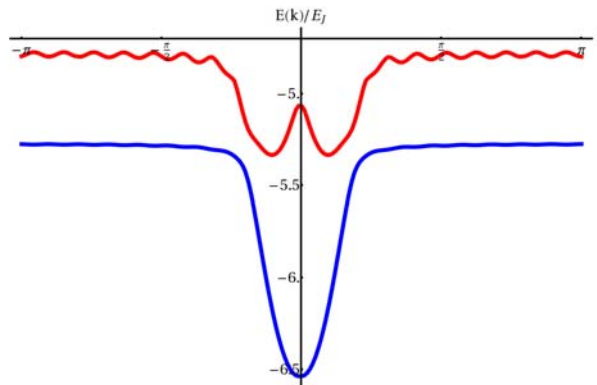


Fig. 3 Dispersion relations of the two lowest bands. The curvature of the lowest energy band in the center of the Brillouin zone shows a strong renormalization of the effective mass.

These results are sufficient to determine (Fig. 4) the amplitude of the persistent current in a ring shaped array with exactly one extra Cooper pair and with no disorder (offset charges). We predict a considerable enhancement of the persistent current, which should be experimentally observable.

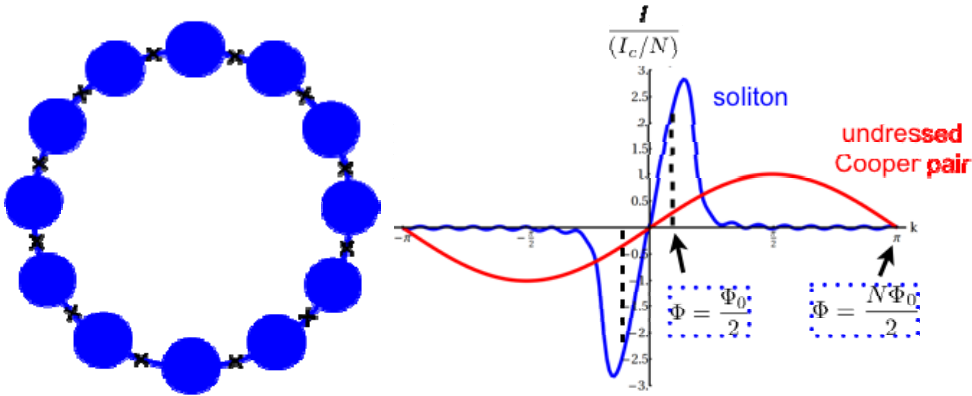


Fig. 4: Persistent current in a ring consisting of  $N$  Josephson junctions as function of the magnetic flux  $\Phi$ . Exactly one extra Cooper pair is present in the ring. Experimentally observable is the dependence in the interval  $\Phi = [-\Phi_0/2, \Phi_0/2]$ , where strong enhancement as compared to an undressed Cooper pair is predicted.

## 1.2 Mean-field theory

In [B3.7:2] we have developed an alternative mean-field theory of charge solitons in terms of the continuous polarization charges  $Q$ . The resulting Lagrangian is of the sine-Gordon type with a  $Q$ -dependent (Bloch) inductance. The theory allows for solitonic solutions with interesting dynamics. In particular these solitons undergo a partial Lorenz contraction. The most important, however, is the fact that the effective mass obtained from the mean-field theory coincides with the one obtained from the tight-binding method in a wide range of parameters (Fig. 5).

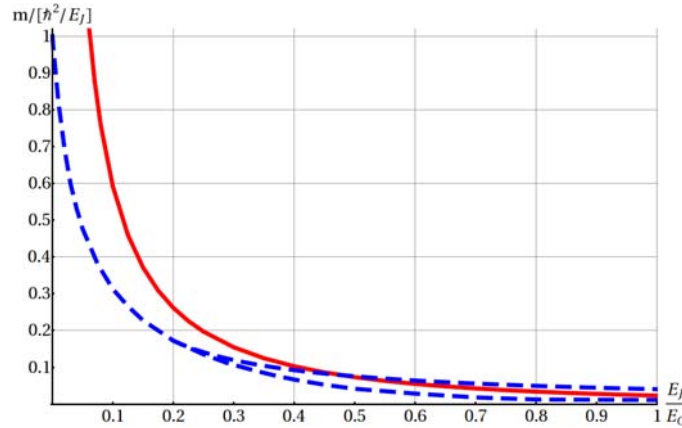


Fig. 5: Red solid curve: soliton mass obtained from the mean-field theory. Blue dashed curve: soliton mass obtained from the tight-binding theory (the doubling of the curve expresses the numerical error of the diagonalization of the tight-binding Hamiltonian).

To describe the transport in more complicated setups, i.e., in an array which is voltage biased at its edges, one has to include the crucial effects of dissipation and disorder. We plan to do so in the future. Yet, our result about the reduction of the effective mass of the solitons is clearly relevant for the theory of charge transport. Indeed the mobility of the charges should increase with decreasing effective mass. Thus, irrespective of the particular transport mechanism one should expect a strong enhancement of the current (conductivity) with increasing  $E_J$ .

## 2 Magnetic Excitations in the Site-Centered Stripe Phase

In [B3.7:3] we have provided a detailed analysis of the initial result of [M. Greiter & H. Schmidt, Phys. Rev. B 82, 144512 (2010)]. We have shown that models of coupled two-leg ladders describing bond-centered stripes cannot explain the magnetic spectrum of  $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$  as the coupling induced by the charge stripes between the ladders is insufficient to induce long-range magnetic order. We have further shown that a model of coupled three-leg ladders describing site-centered stripes accounts accurately for the experimental data.

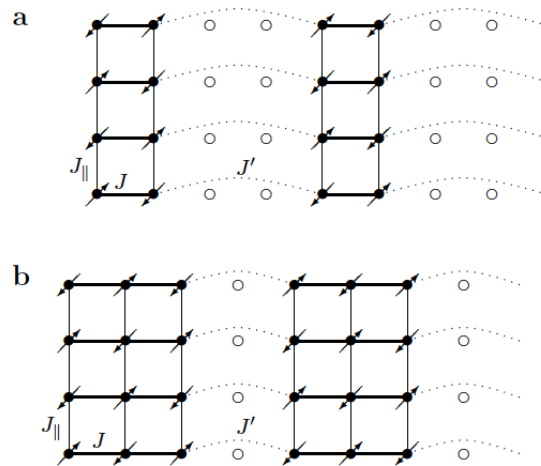


Fig. 6: (a) bond-centered stripes; (b) site-centered stripes.